Using Mutual Information for Exploring Optimal Light Source Placements

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Abstract. Exploring optimal light sources for effectively rendering 3D scenes has been an important research theme especially in the application to computer graphics and visualization problems. Although conventional techniques provide visually plausible solutions to this problem, they did not seek meaningful correlations between proper light sources and their related attribute values. This paper presents an approach to exploring optimal light source placements by taking into account its correlations with such attribute values. Our idea lies in the novel combination of existing formulations by taking advantage of information theory. We first employ the quantized intensity level as the first attribute value together with the conventional illumination entropy so as to find the best light placement as that having the maximum mutual information. Meaningful relationships with viewpoints as the second attribute value are then studied by constructing a joint histogram of the rendered scenes, which is the quantized version of a 3D volume composed by the screen space and intensity levels. The feasibility of the proposed formulation is demonstrated through several experimental results together with simulation of illumination environments in a virtual spacecraft mission.

Keywords: Optimal light source placements · Mutual information · Joint histograms · Scene perception

1 Introduction

The placement of light sources significantly influences on the shape perception of the target 3D object especially in computer graphics and visualization. This is because it directly controls shading effects on its surfaces, and thus provides important depth cues for the 3D scene perception. Several techniques have been proposed to design effective and structured illumination setups, while it is still difficult to assess how much a specific light source can enhance the 3D perception of shape features in the corresponding 3D scene. It is also true that such shading effects also depend on the choice of the viewpoint from which we project the target object onto the projection plane. Indeed, the optimal selection of viewpoints

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itself is another major research theme and a large amount of work has been done so far [16]. Although this issue has been empirically studied in photography and cinematography [10], to the best of our knowledge, only little has been done for understanding the meaningful correlation between the light source placement and viewpoint selection as computational algorithms.

In this paper, we first present an approach to quantitatively evaluating the goodness of the single light source. This is accomplished by incorporating mutual information criteria used for finding optimal viewpoints [6] and extending the conventional work on illumination entropy [7]. This formulation further motivated us to explore the meaningful relationships between the design of light sources and viewpoints. For this purpose, we newly construct a joint histogram with respect to the light sources and viewpoints, by quantizing the 2D screen space and the intensity range occupied by the rendered scene. Mutual information again helps us elucidate the robustness of the selection of light sources in terms of a set of viewpoints in this framework. Several experimental results together with simulation of illumination environments in a virtual spacecraft mission will clarify the effectiveness of the proposed approach.

The remainder of this paper is structured as follows: Sect. 2 conducts a brief survey on previous work relevant to our approach. Section 3 describes our first technical contribution for calculating of the optimal placement of a single light source using mutual information. Section 4 then presents a novel formulation for seeking meaningful correlation between the selection of light sources and viewpoints as our second contribution. Section 5 provides the simulation of illumination environments in a virtual spacecraft navigation as an application study. Finally, we conclude this paper and refer to future work in Sect. 6.

2 Related Work

Light source placement has been an important subject in computer graphics and visualization due to its potential application to enhanced 3D scene perception and lighting design. Gumhold [7] presented an entropy-based measure called illumination entropy, which evaluates the goodness of light source placement by referring to the histogram of the number of pixels with respect to intensity value. The lighting problem has been intensively studied to design interactive systems for placing light sources for illumination of 3D scenes [8,12,15] and volume visualization [24,26]. Perceptual aspects of lighting design have also been researched in recent years [14].

Optimal viewpoint selection has been tackled successfully in the relevant research areas [16]. Again the entropy-based measure has played a significant role for exploring the best viewpoints. Vázquez et al. opened this line of research where they invented the viewpoint entropy measure [21], and later incorporated the concept of view stability and depth maps of the 3D scenes [20]. The formulation of mesh saliency [13], which reveals the distribution of the visual attractiveness of the object surface shapes, has effectively been incorporated into the viewpoint selection problems [4,11]. Several different ideas were also proposed that include clustering similar views on the viewing sphere [25]. Recently, researchers investigated the viewpoint selection problem through machine learning techniques [23] and perceptual studies [17]. Furthermore, the problem was extended to volume visualization, where several techniques have been also developed [1,9,18,19].

The formulation of mutual information has been also incorporated for sophisticating the aforementioned entropy-based formulation. Mutual information can be thought of as a tool for evaluating the degree of dependence between two probability distributions, and has been employed to define the optimal viewpoint with respect to the relative face areas of target 3D objects [6] and quality measure of volume visualization images [2].

Our idea here is to employ the concept of mutual information to retrieve meaningful correlation between the light source placement and other relevant attributes. Although the previous work [22] explored this problem in terms of viewpoint selection, this still remains to be a variant of conventional formulation of illumination entropy [7]. On the other hand, our approach takes advantage of the mutual information criteria [6] to further extend this formulation for computing the optimal placement of a single light source.

3 Optimizing Light Positions Using Mutual Information

This section describes optimal light source placement based on mutual information as the first contribution of this work.

3.1 Illumination Entropy

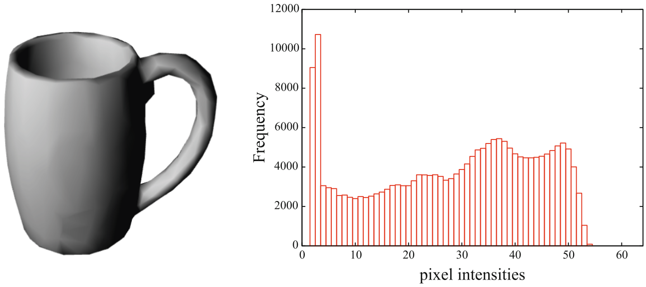
We first explain the illumination entropy devised by Gumhold [7], which is the conventional measure for evaluating the goodness of light source placement. Indeed, this measure employs the Shannon entropy, so as to maximize the uniform distribution of pixel values over the dynamic range of intensity levels. Suppose that $\mathcal{X}$ represents a finite set and $p(x)$ is the probability that $X = x \in \mathcal{X}$, where X is a random variable. The Shannon entropy $H(X)$ is defined as

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x). \tag{1}$$

In this formulation, finding the optimal light source placement amounts to maximally equalizing the occurrence of pixel intensity values in the histogram where the range of intensity values is divided into a specific number of bins. Suppose that p_i represents the normalized occurrence of pixel values that falls into the i-th bin of the histogram. The entropy in Eq. (1) can be rewritten as

$$H = -\sum_{i=1}^{M} p_i \log_2 p_i, \tag{2}$$

where M represents the number of quantization bins in the histogram. Here, we assume that color pixel values (R, G, B) are converted to the corresponding



(a) (b)

Fig. 1. Calculating the illumination entropy: (a) A shaded image of a cup model and (b) its corresponding histogram with respect to the quantized intensity levels.

grayscale value $Y = 0.21262 \cdot R + 0.71514 \cdot G + 0.07215 \cdot B$ [7]. Figure 1 shows an example where the histogram of the quantized intensity levels is computed by scanning the pixel intensity values in the color frame buffer. Note that the number of quantization levels is set to be 64 by default. We will base our new approach on this conventional formulation of the illumination entropy.

3.2 Mutual Information for Light Source Placement

The conventional illumination entropy calculates the Shannon entropy by referring to the probability distribution of the pixels with respect to the intensity value. This leads us to the idea for evaluating meaningful dependence between the light source positions and intensity values. In our approach, we achieved this by introducing the existing framework of mutual information [6] into the formulation of the illumination entropy.

Here, we first assume that we search for the optimal placement of a single light source, and locate its position on the viewing sphere that encloses the target scene. For this purpose, we first take sample points uniformly over the sphere by referring to the vertex positions of a regular icosahedron. Of course, we can increase the number of samples by refining the icosahedron using the common rules of subdivision surfaces. We compute the quality of the light placed at each sample point and find the best placement that maximizes the quality measure.

Now suppose that $\mathcal{X}$ corresponds a finite set of light sources on the viewing sphere and $\mathcal{Y}$ is a set of quantized intensity level. Actually, we consider the information channel between a set of light sources as input and intensity values as output. In our approach, the *relative entropy* (i.e., the *Kullback-Leibler divergence*) $I(x;Y)$ represents how much the occurrences of quantized intensity levels are correlated with respect to the specific light source x_i, and is defined as

$$I(x_i;Y) = \sum_{j=1}^{m} p(y_j|x_i) \log_2 \frac{p(y_j|x_i)}{p(y_j)}, \tag{3}$$

$p(X)$

$p(x_1)$
$\vdots$
$p(x_n)$

$p(Y|X)$

$p(y_1|x_1)$	$\cdots$	$p(y_m|x_1)$
$\vdots$	$\ddots$	$\vdots$
$p(y_1|x_n)$	$\cdots$	$p(y_m|x_n)$

$p(Y)$	$p(y_1)$	$\cdots$	$p(y_m)$

(a)

$\Longleftarrow$ light sources

attribute values

	y_1	$\cdots$	y_n
x_1	$n(x_1, y_1)$	$\cdots$	$n(x_1, y_n)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_n	$n(x_n, y_1)$	$\cdots$	$n(x_n, y_n)$

(b)

Fig. 2. Calculation of relative entropy values. (a) A table of conditional probabilities $p(y_j|x_i)$ where $i = 1, \ldots, n$ and $j = 1, \ldots, m$. (b) The joint histogram with respect to light sources $\{x_i\}$ $(i = 1, \ldots, n)$ and viewpoints $\{y_j\}$ $(j = 1, \ldots, n)$.

where Y is a random variable and $p(y_j)$ is the probability that $Y = y_j$. The *conditional probability* $p(y_j|x_i)$ is the probability of $Y = y_j$ on the condition that $X = x_i$, and can be given by

$$p(y_j|x_i) = \frac{n(x_i, y_j)}{\sum_{j=1}^{m} n(x_i, y_j)}, \tag{4}$$

where $n(x_i, y_j)$ indicates the number of pixels having at the quantization intensity level y_j when the target 3D scene is illuminated from the light source x_i. The *mutual information* between X and Y is defined to be

$$I(X;Y) = \sum_{i}^{n} p(x_i) \sum_{j}^{m} p(y_j|x_i) \log_2 \frac{p(y_j|x_i)}{p(y_j)}. \tag{5}$$

Note that, from $p(x_i)$ and $p(y_j|x_i)$, we can compute the probability $p(y_j)$ as

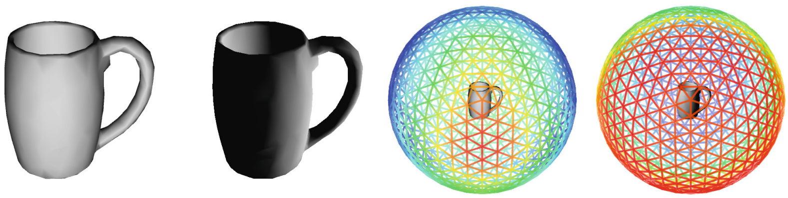
$$p(y_j) = \sum_{i=1}^{n} p(x_i) p(y_j|x_i). \tag{6}$$

Figure 2 shows the relationships among $p(y_j|x_i)$, $p(x_i)$, $p(y_j)$, and $n(x_i, y_j)$.

The above formulation implies that we can identify x_i as the optimal light source placement when the corresponding relative entropy $I(x_i; Y)$ becomes maximum. This is because the corresponding light source produces shading effects that reflect the shape details of the target 3D scene more faithfully, as compared with those having lower entropy values since they are less sensitive to the shape features of the scene. Figures 3(a) and (b) show the comparison between shaded images with the maximum and minimum relative entropy values. This comparison clarifies that the light source with the maximum entropy value can successfully introduce large variation in the distribution of intensity values while that with the minimum entropy produces poor shading effects that spoil the visual quality of the 3D scene.

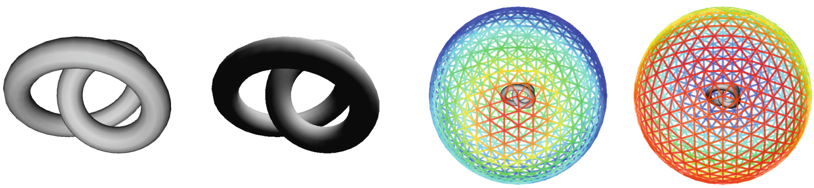
3.3 Experimental Results

We conducted an experiment in order to fully discriminate our approach from the conventional illumination entropy. Figures 3(c) and (d) show such a comparison where we rendered the wireframe sphere according to the quality of the light positions using two approaches. Here, we employ the color gradation that ranges from blue to red according to the quality values. Figure 3(c) presents the distribution of the relative entropy obtained using our approach while Fig. 3(d) corresponds to that of the conventional illumination entropy. Note that in these figures we employ the same viewpoint for both of the two approaches. The comparison undoubtedly indicates that the light positions of high quality are concentrated in the vicinity of the viewpoint in Fig. 3(c) when compared with that in Fig. 3(d). Furthermore, we can confirm that the two directional vectors of the light source and viewpoint span an angle ranges from 15° to 30° in general, which coincides with the fact derived from the perceptual studies in [14]. These results demonstrate that the proposed approach can take into account the perceptual quality of the shaded effects in the 3D scene when selecting the optimal light positions. Note that the same consideration can also be applied to a knot model as shown in Fig. 4.



(a) (b) (c) (d)

Fig. 3. Evaluating illumination effects on the cup model using the mutual information. Results with the (a) maximum and (b) minimum relative entropy. Spherical distribution of the (c) relative entropy and (d) illumination entropy. (Color figure online)



(a) (b) (c) (d)

Fig. 4. Evaluating illumination effects on the knot model using the mutual information. Results with the (a) maximum and (b) minimum relative entropy. Spherical distribution of the (c) relative entropy and (d) illumination entropy. (Color figure online)

4 Correlation Between Light Sources and Viewpoints

Results presented in the previous section clearly suggest that the optimal light placement strongly depends on the choice of the viewpoint. This definitely inspires us to seek meaningful correlation between the light sources and viewpoints. In this section, we present our second contribution of this work, in which we formulated such dependence again by taking advantage of the mutual information.

4.1 Constructing the Joint Histograms

Suppose that this time $\mathcal{X}$ and $\mathcal{Y}$ are finite sets of light sources and viewpoints, respectively, where both are identical with a finite set of samples over the enclosing sphere as described earlier. In this case, the information channel runs between a set of light sources as input and viewpoints as output, or vice versa. This channel actually provides the meaningful correlation between the light source placement and viewpoint selection. Our first task is to compute the joint probability $p(x_i, y_j)\,(i, j = 1, \ldots, n)$, which represents the probability that $X = x_i$ and $Y = y_j$ where n is the number of samples over the sphere. This is accomplished by computing the joint histogram with respect to $\mathcal{X}$ and $\mathcal{Y}$, which can be thought of as a matrix where each (i, j)-th entry $n(x_i, y_j)$ contains the number of specific occurrences when $X = x_i$ and $Y = y_j$ as shown in Fig. 2(b). Normalizing the number of such occurrences at each entry gives us the joint probability as

$$p(x_i, y_j) = \frac{n(x_i, y_j)}{\sum_{i=1}^{n} \sum_{j=1}^{m} n(x_i, y_j)}. \tag{7}$$

Bayes' theorem allows us to compute the conditional probabilities (cf. Fig. 2(a)):

$$p(y_j|x_i) = \frac{p(x_i, y_j)}{p(x_i)} \quad \text{and} \quad p(x_i|y_j) = \frac{p(x_i, y_j)}{p(y_j)}, \tag{8}$$

where $p(x_i) = \sum_{j=1}^{m} p(x_i, y_j)$ and $p(y_j) = \sum_{i=1}^{n} p(x_i, y_j)$. Note that this formulation coincides with Eq. (4), and helps us compute the relative entropy values $I(x_i; Y)$ and $I(y_j; X)$ to assess the dependence of the light source x_i on a set of viewpoints $\{y_j\}$ and the viewpoint y_j on a set of light sources $\{x_i\}$, respectively.

4.2 Counting the Number of Occurrences

We are now ready to define the specific occurrences for constructing the joint histogram of the target 3D scene in terms of light sources and viewpoints. Our idea here is to transform the 3D scene into the frame buffer by projecting it onto the screen space, and then quantize the pixel coordinates and intensity values to compose a 3D volume as shown in Fig. 5. In this figure, the resolution of the 3D volume is $8 \times 8 \times 4$ while in our implementation we employed $32 \times 32 \times 16$ for the resolution. We then count the number of voxels occupied by the quantized

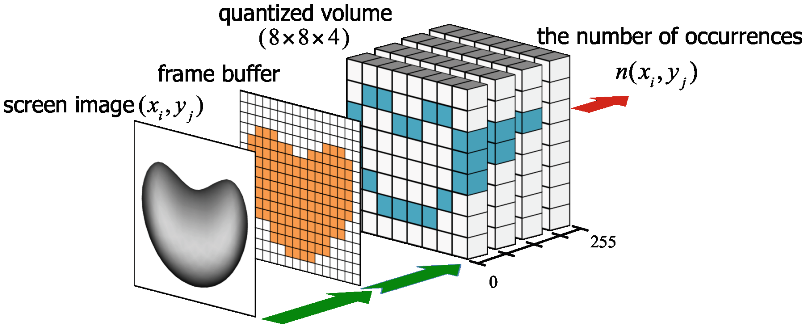


Fig. 5. Transforming the 3D scene into a volume representation by quantizing the screen space and intensity values.

version of the projected scene, as the number of occurrences for the specific light source x_i and viewpoint y_j. Note that we quantize both the 2D screen space and intensity range to evaluate how uniformly the projected scene fills out the quantized 3D volume by counting the number of occupied voxels. Here, we can again identify proper light sources and viewpoints by respectively computing the following relative entropy values:

$$I(x_i; Y) = \sum_{j=1}^{m} p(y_j|x_i) \log_2 \frac{p(y_j|x_i)}{p(y_j)}, \quad \text{and} \tag{9}$$

$$I(y_j; X) = \sum_{i=1}^{n} p(x_i|y_j) \log_2 \frac{p(x_i|y_j)}{p(x_i)}. \tag{10}$$

4.3 Experimental Results

We still need to provide a specific interpretation of the light sources and viewpoints that have the minimum or maximum relative entropy values. If a light source has the minimum relative entropy in Eq. (9), it will give a consistent illumination effects regardless of the selection of the viewpoints. If it has the maximum relative entropy, its illumination effects will drastically change according to the choice of the viewpoints. Figure 6 presents the variation of the relative entropy Eq. (9) of the cup model and its representative snapshots with the light sources having the minimum and maximum relative entropy values. Note that representative snapshots are taken from 12 viewpoints that coincide with the 12 vertices of the regular icosahedron as described earlier. Good light sources appear diagonally above the cup model to better illuminate its inside as shown in Fig. 6(a). Furthermore, the light source of the minimum relative entropy value produces consistent illumination effects as shown in Fig. 6(b) while that of the maximum entropy value yields unstable illumination that drastically differs among the viewpoints as shown in Fig. 6(c).

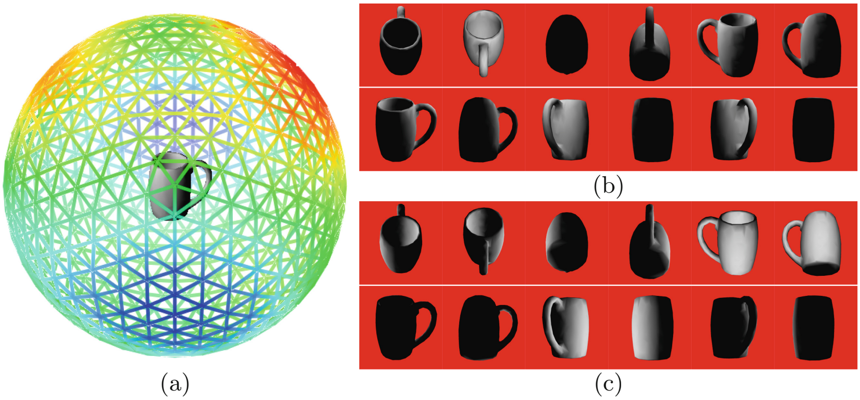


Fig. 6. Light source placement for a cup model. (a) Distribution of the relative entropy values for the light sources over the sphere. Representative snapshots with the light sources with the (b) minimum relative entropy and (c) maximum relative entropy.

As a side effect, on the other hand, we can potentially take advantage of the viewpoints having the minimum and maximum relative entropy values. The viewpoint having the minimum relative entropy commonly adds important 3D depth cues in the corresponding projected images, while that of the maximum entropy value cannot fully guarantee the quality of the projected images with a specific subset of light sources. Figure 7 clearly demonstrates this trend.

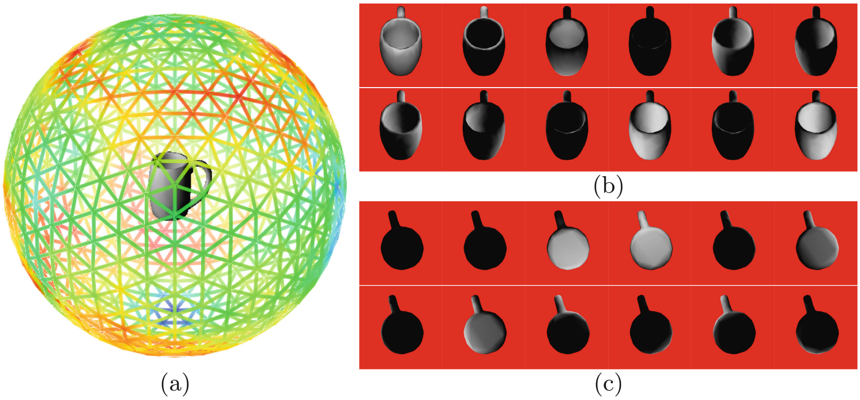
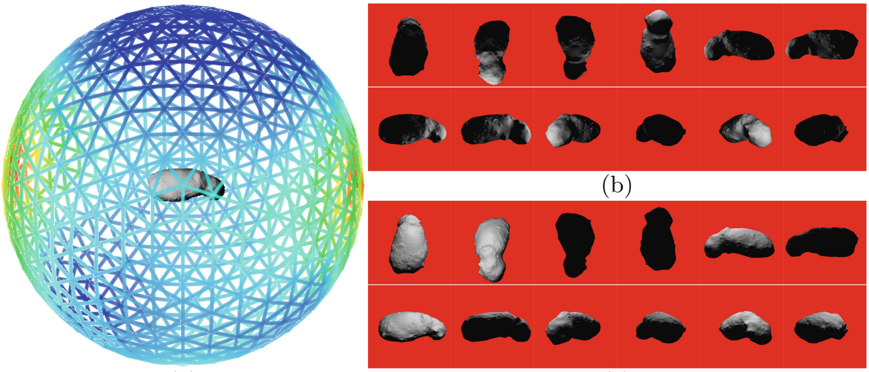


Fig. 7. Viewpoint selection for a cup model. (a) Distribution of the relative entropy values for the viewpoints over the sphere. Representative snapshots with the viewpoints with the (b) minimum relative entropy and (c) maximum relative entropy.

5 Simulating Illumination Effects for Spacecraft Missions

In this section, we simulate illumination effects on an asteroid to seek the better control of a virtual spacecraft mission. A well-known spacecraft called "Hayabusa" was launched by Japan Aerospace eXploration Agency (JAXA) in 2003, with the objective to land an asteroid called "Itokawa" and collect samples of asteroid materials, and successfully returned to the earth with the samples in 2010. In this spacecraft mission, it was very important to reconstruct the shape details of Itokawa for analyzing its spatial motion and formation process [3,5]. For recovering the 3D shape of the asteroid, we usually take photograph images from the spacecraft and investigate them to reconstruct its 3D shape, while its illumination conditions are severely constrained especially in the space. This is due to the fact that we have only one light source (i.e. the sun), and the degrees of freedom of the spacecraft navigation are quite low due to the influence of gravity and limited amount of fuel. Therefore, precomputing the illumination conditions is indeed beneficial provided that a rough model of the target asteroid is available. This also lets us effectively navigate the spacecraft in the sense that we can maximally search for specific light conditions and viewpoints to extract more information about the details of the asteroid shape.

In our experiment, we took the 3D shape of Itokawa [5] as an input, and computed the relative entropy values of the light sources to evaluate which light conditions are effective for the mission. Figure 8 shows the distribution of the relative entropy values of the light sources over the sphere in terms of the viewpoints. Again, in this case, minimizing the relative entropy allows us to find an optimal light position that produces steady illumination effects regardless of the choice of the viewpoints. On the other hand, the light source with the maximum relative entropy often fails to exhibit shape details of the asteroid when we see



(a) (b) (c)

Fig. 8. Light source placement for the Itokawa model. (a) Distribution of the relative entropy values for the light sources over the sphere. Representative snapshots with the light sources with the (b) minimum relative entropy and (c) maximum relative entropy.

from different viewpoints. In this way, our approach for finding optimal light conditions is effective in controlling the illumination effects, which are severely constrained often in the spacecraft missions.

6 Conclusion

This paper has presented an approach to effectively evaluating the illumination quality produced by a single light source. Our technical contribution is two fold; the first is to formulate the optimal placement of a light source in terms of the distribution of the intensity values and the second is to seek the optimal illumination condition that is robust against the choice of the viewpoints. Mutual information has been introduced to elucidate the optimal placement of a light source in the sense that the associated illumination condition is consistent among pixel intensity values and a possible set of viewpoints. We presented several experimental results together with the simulation of illumination effects in the virtual spacecraft mission, to demonstrate the applicability of our approach.

Our future work includes more rigorous understanding of the relative entropy for the attribute values. In practice, the relative entropy in Eq. (10) enables us to find optimal viewpoints that are robust against the choice of light sources as shown in Fig. 7. Nonetheless, this process sometimes fails because the minimum entropy value can correspond to the viewpoint yielding a smaller projected area of the target 3D scene to avoid the influence by the choice of light sources. Incorporating multiple light sources into our formulation is another interesting research theme. Extending the proposed assessment of illumination effects to volume visualization is also left as our future work. More rigorous perceptual studies for the assessment of our approach still remain to be made.

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