Adaptive Blending of Multiple Network Layouts for Overlap-Free Labeling

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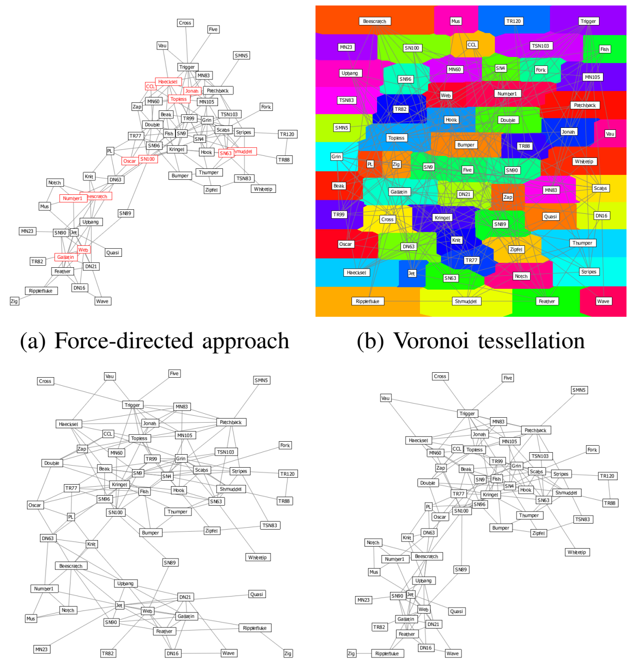
***Abstract*—Conventional force-directed algorithms are known as a common approach to aesthetically drawing networks while they still suffer from self-overlaps especially when the network nodes are annotated with text labels. Incorporating space partitioning techniques including Voronoi tessellation are often effective to spare enough space around each node while this may incur different artifacts such as unexpectedly long edges and edge overlaps. This paper presents an approach to resolving overlaps among node labels by adaptively blending multiple layout forces applied to the respective network nodes. This is accomplished by extending our previous approach for transforming the force-directed layout into that obtained through the centroidal Voronoi tessellation. Our technical contribution lies in a novel algorithm for smoothing blending ratios associated with the network nodes so that we can adaptively explore the reasonable balance between the two layouts independently for each node. Experimental results will present that our new approach can produce well-balanced distribution of node labels while maximally avoiding the aforementioned unwanted visual artifacts.**

***Keywords*-Adaptive layout blending; force-directed graph layout; Centroidal Voronoi tessellation; smoothing;**

I. Introduction

Understanding mutual relationships among individuals/entities motivates us to represent them as networks, which usually consist of nodes as the individuals/entities and edges as their connections. Furthermore, the network nodes often have text labels as annotations that help us to instantly understand what each node represents in the relationships. Force-directed algorithms are a common approach to aesthetically laying out such networks, while they still suffer from visual clutter arising from overlaps among annotation labels. This encourages many researchers to devise new techniques for effectively suppressing mutual overlaps among such annotation labels.

One of the promising ideas is to incorporate space partitioning techniques into the conventional force-directed algorithms for deliberately yielding sufficient labeling space around each node. Voronoi tessellation is a popular approach for that purpose, while this may again incur unwanted artifacts in the network layouts such as unequal lengths of edges and sharp angles between adjacent edges. This suggests that we should maximally retain aesthetic network layouts obtained through the force-directed algorithms especially when we introduce additional space for annotation labels around the nodes. Actually, it is significant to explore a reasonable compromise between the force-directed network layout and uniform partitioning of the screen space.



(a) Force-directed approach (b) Voronoi tessellation (c) Previous approach (d) Our approach

Figure 1. Network layouts of social relationships among dolphins [1]. (a) A force-directed layout. (b) A layout based on centroidal Voronoi tesselation. (c) A layout produced by the previous approach [2]. (d) A layout generated by our approach. Our approach yields an aesthetic layout of the annotated dolphin network by exploring a reasonable compromise between the force-directed layout (a) and space partitioning based on centroidal Voronoi tessellation (b), which is much closer to the force-directed layout than that produced by the previous approach (c). Annotation labels having overlaps with others are drawn in red.

In this paper, we extend our previous approach [2] so that we can aggressively seek a better compromise between the two layouts for avoiding overlaps among annotation labels associated with the network nodes. The primary idea behind our approach is to adaptively blend multiple forces exerted by these two layouts individually for each node, and thus the proposed algorithm distinctly differs from the previous approach that applies the same blending ratio uniformly to all the network nodes. In practice, we accomplish this by smoothly blending the ratio of the two forces at each node, with those of other nodes within its neighborhood.

Experimental results will demonstrate that our proposed approach successfully localizes the space allocation required for each node, and provides us with relatively compact layout in the sense that annotation labels are tightly fit to the network while avoiding their unwanted overlaps.

Figure 1 presents such an example where our approach can yield a visually plausible layout of an annotated network (Figure 1(d)) by exploring a reasonable compromise between the force-directed layout (Figure 1(a)) and space partitioning based on the Voronoi tessellation (Figure 1(b)). As shown in the figure, our approach will compose a compact layout that is relatively close to the original force-directed layout. Furthermore, we can maximally retain the aesthetic criteria induced from the conventional force-directed layout while suppressing undesirable artifacts such as unequal lengths of edges and small angles between adjacent edges, which often arise from space partitioning schemes based on the Voronoi tessellation (Figure 1(c)).

This paper is structured as follows: Section II conducts a brief survey on relevant studies including network visualization and label annotation. Section III reviews our previous approach, which serves as a basis for our new algorithm for drawing annotated networks. Section IV details primary technical contributions of this work, including schemes for smoothing blending ratios of multiple layout forces individually at the network nodes. Experimental results are presented to demonstrate the feasibility of our approach in Section V, which is followed by conclusion and future work in Section VI.

II. Related Work

In this section, we provide a brief survey on related work, including network visualization and label annotation.

A. Network Visualization

Network visualization is one of the major research themes and has commonly been employed for visually analyzing the complicated networks such as friendships in communities, co-authorships among researchers, product co-purchasing data, etc [3]. With the advancement of network visualization techniques, improving the network readability has become an important technical challenge and a large number of layout algorithms have been proposed for that purpose.

In general, force-directed algorithms [4], [5] simulate dynamics of network behavior and serve as the primary means of finding aesthetic layouts of such networks. They usually model the network nodes and edges as particles with electronic charges and coil springs, respectively, and optimize the network layout by finding the equilibrium of repulsive and drawing forces arising from Coulomb's law and Hooke's law at the respective nodes. Empirically, the equilibrium state leads to an aesthetic layout of the network in the sense that it minimizes unequal distribution of nodes and unwanted crossings among the edges [6].

B. Label Annotation

Although the aforementioned force-directed algorithms provide visually-pleasing network layouts, they cannot directly remove mutual overlaps among text annotation labels if they are assigned to the network nodes. To solve this problem, more techniques are developed for explicitly controlling the space allocation around the annotation labels [7], [8].

Space partitioning techniques also help us to spare sufficient space for placing annotation labels while respecting the aesthetic layout obtained through the conventional force-redirected algorithms. Among them, Voronoi tessellation is widely used and facilitates us to allocate reasonable territorial regions around the given sample points within the screen space. For example, Pulo [9] introduced the concept of recursive Voronoi tessellation instead of the quadtree representation, so that he could enhance the scalability of the network visualization based on the force-directed approach. Wu et al. [10] took advantage of the Voronoi tessellation for placing annotation labels around schematized metro networks. Brivio et al. [11] incorporated centroidal Voronoi tessellation in order to visualize image sets compactly.

Indeed, the centroidal Voronoi tessellation has been integrated into the convectional force-directed approach in order to aggressively spare more space around each network node, which effectively allows us to suppress the overlap among the text annotation labels. Lyons et al. [12] iteratively applied additional forces exerted by the centroidal Voronoi tessellation until its similarity to the original force-directed layout is preserved within a certain tolerance. Gansner and North [13] introduced influence obtained from the centroidal Voronoi tessellation as a post-processing until the overlap among annotation labels is fully resolved. Nonetheless, these approaches compose the Voronoi tessellation once at an early stage during the network visualization to retain the original network layout, while they require relatively large extra screen space to avoid unwelcome label overlaps. Nevertheless, Ishida et al. [2] combined forces of the conventional force-directed approach and centroidal Voronoi tessellation, and aggressively excluded label overlaps by properly controlling the blending ratio between the two forces. This approach allows us to eliminate unwanted overlap among annotation labels within a relatively small screen space at the cost of breaking the original layout to some extent, but may also incur other artifacts such as unequal edge lengths and small angles spanned by adjacent edges since the distribution of network nodes is closer to a grid-like layout. In this paper, we thus extend our work [2] for reducing such visual artifacts by adjusting the blending ratios of the two layout forces adaptively at each node.

III. Space Partitioning for Annotating Networks

As mentioned earlier, the proposed approach can be thought of as an extension of our previous work [2], where we explored a weighted sum of the network layouts obtained by the force-directed approach and centroidal Voronoi tessellation. This allows us to avoid unwanted overlaps among annotation labels and unexpected sharp angles between adjacent edges. The section provides fundamentals of this work before explaining our new approach in the next section.

A. Forces Produced by the Two Layouts

The formulation developed in the previous work [2] is a hybrid approach that it combines the layout forces obtained through the conventional force-directed approach and space partitioning approach based on the centroidal Voronoi tessellation. As described in Section II, the force-directed approach explores an equilibrium state of the network based on the assumption that every node has an electronic charge and every edge has a coil spring. Thus, we can calculate the final layout by minimizing the sum of the forces $F_f(\boldsymbol{v}_i)$ ($i \in V$), where $F_f(\boldsymbol{v}_i)$ applied to node $\boldsymbol{v}_i$ is defined as:

$$\begin{aligned} F_f(\boldsymbol{v}_i) &= F_r(\boldsymbol{v}_i) + F_d(\boldsymbol{v}_i), \quad \text{where} \\ F_r(\boldsymbol{v}_i) &= -\sum_{k \in V - \{i\}} \frac{k_r(\boldsymbol{v}_k - \boldsymbol{v}_i)}{|\boldsymbol{v}_i - \boldsymbol{v}_k|^2} \text{ and} \\ F_d(\boldsymbol{v}_i) &= \sum_{j \in A_i} k_d(|\boldsymbol{v}_i - \boldsymbol{v}_j| - l_0)(\boldsymbol{v}_j - \boldsymbol{v}_i). \end{aligned} \tag{1}$$

Note that $F_r(\boldsymbol{v}_i)$ and $F_d(\boldsymbol{v}_i)$ are the repulsive and drawing forces exerted to $\boldsymbol{v}_i$, V represents the index set of the network nodes, A_i corresponds to an index set of nodes adjacent to $\boldsymbol{v}_i$, and $V - \{i\}$ is an index set of all the nodes except for i. In addition, k_r and k_d are the coefficients for the repulsive and drawing forces and l_0 is the free length of the coil spring. Here, $l_0 = \sqrt{S/|V|}$ by default, where S and $|V|$ represent the total area of the screen and the size of V, respectively. Figure 1(a) represents a network layout obtained by the force-directed approach.

On the other hand, we can obtain the uniformly distribution of the network nodes by referring to the centroidal Voronoi tessellation as shown in Figure 1(b). Note that the centroidal Voronoi tessellation is obtained by iteratively computing the Voronoi diagram from the current set of network nodes and moving them toward the centroids (i.e., gravity centers) of their corresponding Voronoi cells until the network layout converges. In this computation, the force $F_v(\boldsymbol{v}_i)$ that is applied to the node $\boldsymbol{v}_i$ can be given by

$$F_v(\boldsymbol{v}_i) = C(\boldsymbol{g}(\boldsymbol{v}_i) - \boldsymbol{v}_i), \tag{2}$$

where $\boldsymbol{g}(\boldsymbol{v}_i)$ is the centroid of the corresponding Voronoi cell and C is a constant that controls the motion of $\boldsymbol{v}_i$.

B. Blending the Two Forces

In the previous work, the network layout is obtained first by using the force-directed approach only and then gradually integrating the layout forces derived from the centroidal Voronoi tessellation. This hybrid approach is achieved by adjusting the force $F_h(\boldsymbol{v}_i)$ applied to each node $\boldsymbol{v}_i$ as:

$$F_h(\boldsymbol{v}_i) = (1 - \alpha)F_f(\boldsymbol{v}_i) + \alpha F_v(\boldsymbol{v}_i), \tag{3}$$

where the blending ratio α is raised step by step to incorporate more influence from the centroidal Voronoi tessellation, until overlap among the text labels is completely resolved.

The total area of the label overlap can be computed as follows. Suppose that the node $\boldsymbol{v}_i$ has the rectangular text label where its width and height are represented by w_i and h_i, respectively. Let us denote by w_{ij} and h_{ij} the width and height of the rectangular area shared by the text labels at the nodes $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$. We can obtain these values as

$$\begin{aligned} w_{ij} &= \max\{0, \min\{w_i/2 + w_j/2 - |x_{ij}|, w_i, w_j\}\} \text{ and} \\ h_{ij} &= \max\{0, \min\{h_i/2 + h_j/2 - |y_{ij}|, h_i, h_j\}\}, \end{aligned} \tag{4}$$

where x_{ij} and y_{ij} are the x- and y-components of the vector $\boldsymbol{v}_j - \boldsymbol{v}_i$, respectively.

In our previous work, we prepared only one unique blending ratio α for the entire annotated network. Although increasing α allows us to effectively spare more space around the network nodes by referring to the centroidal Voronoi tessellation and thus suppress unwanted visual clutter arising from label overlaps, this setup incurs unexpected visual artifacts such as unequal edge lengths and nearly overlapped edges unexpectedly. We will describe how we solve them by individually adjusting the blending ratio at each network node in Section IV.

C. Computing the Centroidal Voronoi Tessellation

We employed a hardware-assisted algorithm for accelerating the computation of Voronoi tessellation [14], since we compute the space partition of the screen space every time when updating the positions of the network nodes. In this algorithm, we can identify the territorial cell associated with each node by placing a cone centered at that node. Here, each cone is arranged in a way that its axis is perpendicular to the 2D screen space. Thus, we can naturally obtain the Voronoi partitioning of the screen space by assigning difference colors to the cones and looking at the screen from the top. This algorithm is fast since we only draw 3D cones using commonly available 3D graphics hardware. For evaluating the forces exerted by the centroidal Voronoi tessellation, we collect pixels within each Voronoi cell by scanning their colors and finding the corresponding centroid as the average position of the pixels having the corresponding color. This instantly gives us the forces arising from the space partitioning and thus an updated network layout.

However, in this specific application, selecting the ordinary Euclidean distance metric is not the best because we usually place rectangular labels on the screen. In this previous work, we selected the Chebyshev distance metric instead in order to spare rectangular territorial cells for each network node. Note that the Chebyshev distance between the two nodes $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$ is defined to be:

$$d(\boldsymbol{v}_i, \boldsymbol{v}_j) = \max(|x_1 - x_2|, |y_1 - y_2|). \tag{5}$$

This can be immediately implemented by placing rectangular pyramids instead of cones in the hardware-assisted algorithm, where the two diagonals of each rectangular basement are exactly aligned with x- and y-axes, respectively. Furthermore, for taking into account the aspect ratio of each text label, we also adjust the lengths of the two diagonals in such a way that their aspect ratio is identical with that label. Figure 1(b) shows the space partitioning obtained through the enhancements we have described so far. This successfully spares the labeling space that properly fits to the shape of the annotation label around each network node.

IV. Adaptively Blending Network Layouts

Our previous work [2] provides an overlap-free layout of an annotated network by allocating sufficient labeling space for each node, while it often incurs redundant space as shown in Figure 1(c) since it employs one unique ratio for blending the two network layouts. This often degrades the quality of the network layout because the work cannot fully retain the aesthetics obtained through the force-directed approach. This observation leads to the idea of changing the blending ratios respectively at the network nodes, which will be detailed in the remainder of this section.

A. Laplacian Smoothing for Adjusting Blending Ratios

The problem of the previous work is to explore the global compromise between the force-directed layout and that obtained from the centroidal Voronoi tessellation through the control of the single blending parameter. In practice, it is better to find a local compromise so that we can adaptively spare labeling space around each node only if its corresponding text label has overlap with other neighboring labels. This implies that we have to plausibly propagate such space allocation request from that node to its neighbors, which is equivalent to searching for smooth transition of the blending ratios over the screen space.

The overall process of the proposed approach can be summarized as follows: First of all, we compute the initial network layout only with the force-directed approach, in the same way as we did in the previous work. We then adaptively change the blending ratio of each network node to explore its local compromise between the two layouts. Let us denote the blending ratio of $\boldsymbol{v}_i$ by α_i. The adaptive blending of the two layouts is accomplished by increasing α_i by a small amount (0.01 by default in our implementation) if its corresponding text label still has spatial overlap with other neighboring labels, and then smoothing out α_i by referring to those of its neighbors. Note that the blending ratio α_i is initialized to 0.0 at the beginning. This process is iteratively repeated until we can fully excluded label overlaps.

As for the step for finding the smooth transition of the blending ratios, the most common approach is to apply the

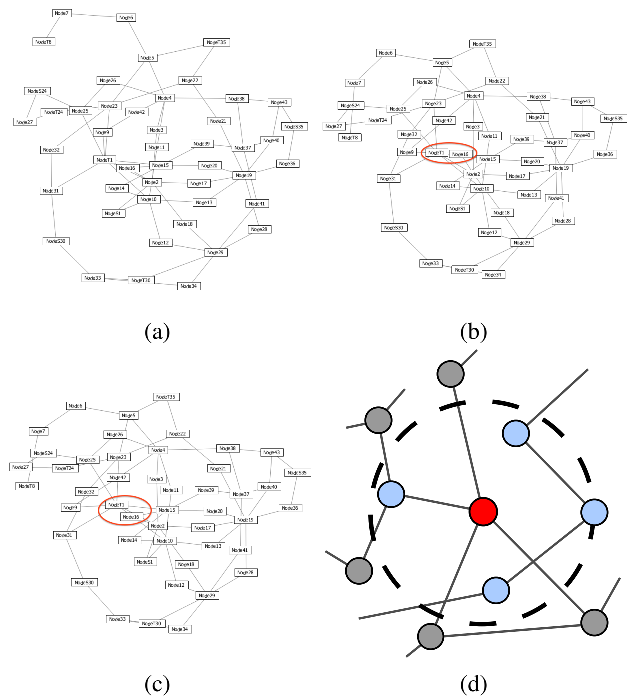


Figure 2. Difference between the network layouts obtained by (a) Laplacian smoothing with the Euclidean distance metric, (b) Taubin's two-step smoothing with the Euclidean distance metric, and (c) Taubin's two-step smoothing with the Chebyshev distance metric. (d) Definition of the k-neighbors around the network node.

Laplacian smoothing, which updates α_i to α'_i by

$$\alpha'_i = \sum_{j \in N_i} \alpha_j / |N_i|, \tag{6}$$

where N_i is an index set of the neighbors around $\boldsymbol{v}_i$ and $|N_i|$ is its size. This amounts to replace the blending ratio at $\boldsymbol{v}_i$ with the average of the ratios at its neighbors. Figure 2(a) shows a result obtained by this Laplacian smoothing.

B. Two-Step Smoothing for Adjusting Blending Ratios

In practice, the Laplacian smoothing provides us with visually pleasing layouts while they still contain unnecessary space around network nodes especially when they are sparsely arranged. This is because the distribution of the blending ratios at the network nodes are more likely to be even especially when we applied the Laplacian smoothing a large number of times.

In order to properly smooth out the spatial distribution of blending ratios while preserving its original undulation, our new approach employed Taubin's two-step smoothing [15]. The advantage of this smoothing operation is that it works as an approximate low-pass filter and thus never incurs unwanted shrinkage problems as the conventional Gaussian filters do. Taubin's smoothing consists of alternating shrinking and expanding steps, which can be formulated as:

$$\begin{aligned} \alpha'_i &= \alpha_i + \lambda\{(\sum_{j \in N_i} \alpha_j / |N_i|) - \alpha_i\} \quad \text{and} \\ \alpha''_i &= \alpha'_i + \mu\{(\sum_{j \in N'_i} \alpha'_j / |N'_i|) - \alpha'_i\}. \end{aligned} \tag{7}$$

where $0 < \lambda < -\mu$. Here, the first equation corresponds to the shrinking step that updates α_i to α'_i while the second equation represents the expanding step for replacing α'_i with α''_i. Note that $|N_i|$ and $|N'_i|$ represent the sizes of the neighbors around $\boldsymbol{v}_i$ at each step. See [15] for the choice of λ and μ. By applying this two-step smoothing, we can obtain the network layout that is more compact and closer to the force-directed layout. Figure 2(b) presents such a result obtained by this Taubin's two-step smoothing. In this layout, the network nodes are more tightly connected than that in Figure 2(a) while successfully excluding label overlaps.

C. Selecting Neighbor Nodes

Our final task is to select the neighbors appropriately around each network node. In this case, we search for spatially close neighbors rather than topologically adjacent ones since we would like to avoid mutual overlaps between text labels that are next to each other. One possible solution is to collect neighbors within a specific radius of the target node. In practice, we tested this solution while in this case the number of neighbors varies and thus sometimes becomes zero, which means that the target node can be isolated from others when smoothing out the blending ratios. What we choose here is to collect the k nearest neighbors at each network node, as shown in Figure 2(d). In our implementation, we employed $k = 4$ since this setup empirically worked well. we again employed the Chebyshev distance as our distance metric. This effect is elucidated through the comparison between Figures 2(b) and (c). Here, the Chebyshev distance is slightly better since it empirically produces the space around the horizontally elongated labels.

V. Experimental Results

Our prototype system has been implemented on a desktop PC with 3.5 GHz 6-Core Intel Xeon E5 CPU and 32GB RAM. The source code was written in C++ using OpenGL for graphics rendering and Qt for interface. To confine all the annotation labels, we also exerted forces from the boundary of the screen space to the nodes if they are annotated with the text labels. We employed datasets provided by the Graphviz (http://www.graphviz.org/) in our experiment. In captions for Figures 3 and 4, we denote the computation time and percentage of overlap with respect to the total area of labels by T and O, respectively.

Figure 3 provides visual comparison among three layouts produced by the conventional force-directed approach, the present approach, and the previous work [2] for the medium-sized network "b124." This comparison clarifies that the proposed approach generates a network layout that looks similar to the initial force-directed one while avoiding mutual overlaps among annotation labels. Figure 4 demonstrates layouts of a large-sized network "b143," where our new approach can locally adjust the labeling space around each network node by taking advantage of available screen space. As a limitation, there still exists an extreme case that the label overlaps could be locally blocked due to insufficient space for label placement around the boundary regions.

VI. Conclusion

This paper has presented an approach for exploring a reasonable compromise between the force-directed layout and that based on the centroidal Voronoi tessellation when laying out annotated networks. The present approach facilitates us to exclude unacceptable overlaps between text labels associated with the network nodes while maximally retaining the aesthetics initially provided by the force-directed approach. This is accomplished by adaptively adjusting the blending ratio between the two layouts at each node according to the requirement for labeling space first and then smoothing it out with those of its spatial neighbors through the shrinkage-free filter.

More sophisticated control over the blending ratios at the respective network nodes should be tackled to effectively handle large-scale annotated networks. Incorporating additional forces for better readability of the layouts is also an interesting theme for future research.

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References

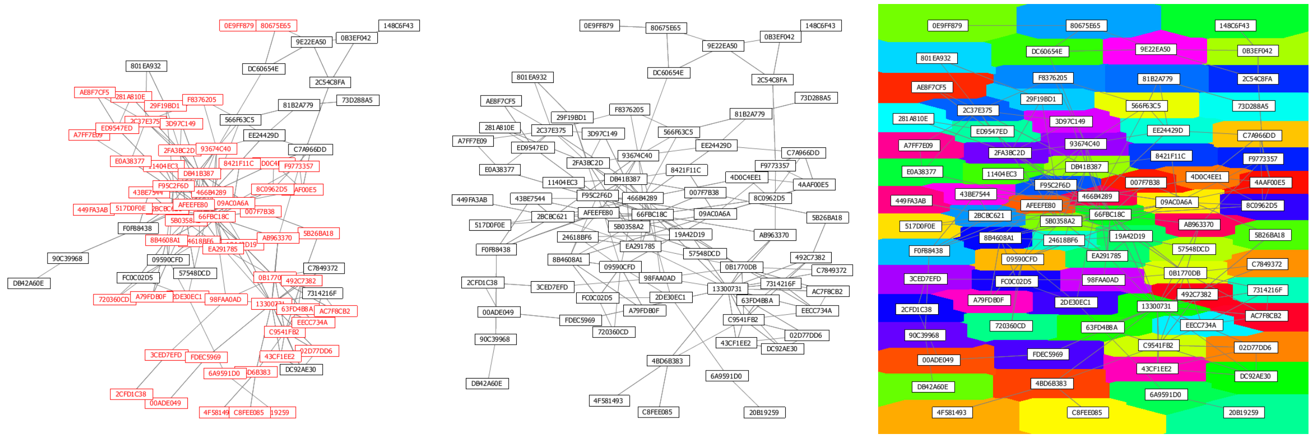
[1] D. Lusseau, K. Schneider, O. J. Boisseau, P. Haase, E. Slooten, and S. M. Dawson, "The bottlenose dolphin community of doubtful sound features a large proportion of long-lasting associations," *Behavioral Ecology and Sociobiology*, vol. 54, pp. 396–405, 2003.

[2] R. Ishida, S. Takahashi, and H.-Y. Wu, "Interactively uncluttering node overlaps for network visualization," in *Proceedings of the 19th International Conference on Information Visualization (IV2015)*, 2015, pp. 200–205.

[3] J. Heer and D. Boyd, "Vizster: Visualizing online social networks," in *Proceedings of IEEE Symposium on Information Visualization 2005 (InfoVis 2015)*, 2005, pp. 32–39.

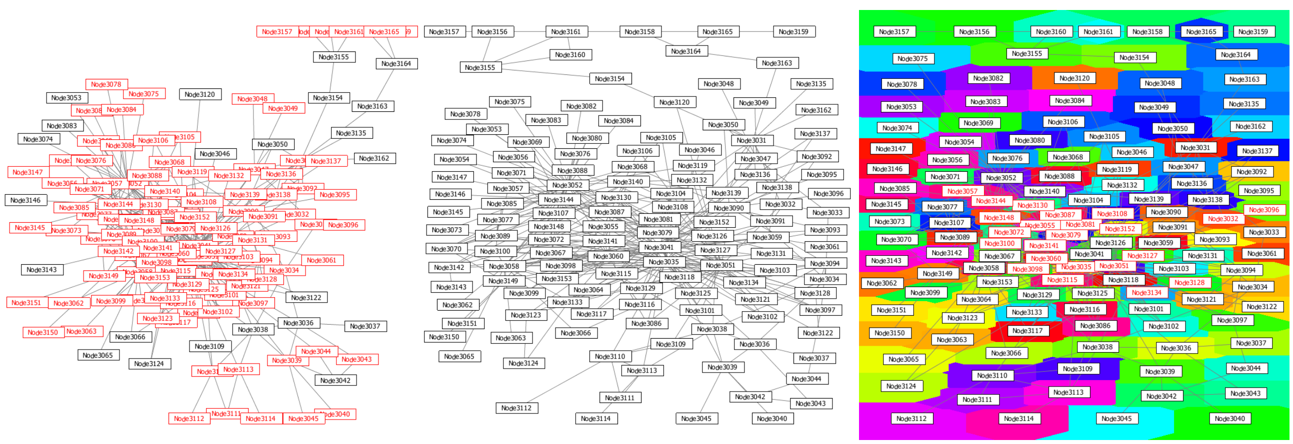
[4] P. Eades, "A heuristic for graph drawing," in *Proceedings of the Congressus Numerantium*, vol. 42, 1984, pp. 149–160.

[5] T. M. J. Fruchterman and E. M. Reingold, "Graph drawing by force-directed placement," *Software - Practice and Experience*, vol. 21, no. 11, pp. 1129–1164, 1991.



(a) (b) (c)

Figure 3. Visualizing the “b124” network data ($|V| = 79, |E| = 281$). (a) A layout obtained using the force-directed algorithm (T = 16.7 sec, O = 5.9%). (b) A layout obtained using our proposed approach (T = 13.1 sec, O = 0.0%). (c) A layout obtained till an equilibrium state using our previous approach where each Voronoi cell is rendered in a difference color (T = 19.8 sec, O = 0.0%).



(a) (b) (c)

Figure 4. Visualizing the “b143” network data ($|V| = 135, |E| = 366$). (a) A layout obtained using the force-directed algorithm (T = 48.6 sec, O = 14.1%). (b) A layout obtained using our proposed approach (T = 81.7 sec, O = 0.0%). (c) A layout obtained till an equilibrium state using our previous approach where each Voronoi cell is rendered in a difference color (T = 72.2 sec, O = 0.4%).

[6] H. C. Purchase, “Metrics for graph drawing aesthetics,” *Journal of Visual Languages and Computing*, vol. 13, no. 5, pp. 501–516, 2002.

[7] T. Dwyer, K. Marriott, and P. J. Stuckey, “Fast node overlap removal,” in *Proceedings of the 13th International Symposium on Graph Drawing*, ser. Springer LNCS, vol. 3843, 2005, pp. 153–164.

[8] E. R. Gansner and Y. Hu, “Efficient node overlap removal using a proximity stress model.” in *Graph Drawing*, ser. Springer LNCS, vol. 5417, 2008, pp. 206–217.

[9] K. J. Pulo, “Recursive space decompositions in force-directed graph drawing algorithms,” in *Proceedings of Asia-Pacific Symposium on Information Visualisation (APVis ’01)*, 2001, pp. 95–102.

[10] H.-Y. Wu, S. Takahashi, C.-C. Lin, and H.-C. Yen, “Voronoi-based label placement for metro maps,” in *Proceedings of the 17th International Conference on Information Visualisation (IV2013)*, no. 96–101, 2013.

[11] P. Brivio, M. Tarini, and P. Cignoni, “Browsing large image datasets through Voronoi diagrams,” *IEEE Transactions on Visualization and Computer Graphics*, vol. 16, no. 6, pp. 1261–1270, 2010.

[12] K. A. Lyons, H. Meijer, and D. Rappaport, “Algorithms for cluster busting in anchored graph drawing,” *Journal of Graph Algorithms and Applications*, vol. 2, no. 1, pp. 1–24, 1998.

[13] E. R. Gansner and S. C. North, “Improved force-directed layouts,” in *Proceedings of the 6th International Symposium on Graph Drawing*, ser. Springer LNCS, vol. 1547, 1998, pp. 364–373.

[14] K. E. Hoff, III, J. Keyser, M. Lin, D. Manocha, and T. Culver, “Fast computation of generalized Voronoi diagrams using

graphics hardware," in *Proceedings of SIGGRAPH '99*, 1999, pp. 277–286.

[15] G. Taubin, "Curve and surface smoothing without shrinkage," in *Proceedings of the 5th International Conference on Computer Vision (ICCV'95)*, 1995, pp. 852–857.